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Complexity of cellular manufacturing systems based on entropy models[†]

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Abstract

Even structurally simple manufacturing systems can be operationally complex. This operational complexity can be colloquially defined as the uncertainty associated with managing the dynamic variations, in time or quantity, across information and material flows at the manufacturing systems interface. This paper proposes a means of measuring the information demands placed on cellular manufacturing systems, as a result of this uncertainty. A utility function for complexity is proposed according to the relationships between the complexity and utility in a manufacturing system and the underlying trend that the system becomes more and more complex in an ever-changing environment is analyzed. This paper mathematically models the static entropy and the dynamic entropy of cellular manufacturing systems from an information-theoretic perspective. A unique feature of this measure is that it captures, in relative terms, the expected amount of information required to describe the state of the system. The measure provides flexibility in the scope and detail of analysis. Finally, an example is used to demonstrate the validity of the proposed methodology.

Keywords: Cellular manufacturing systems; Complexity; Entropy model; Utility

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1. Introduction

The modern manufacturing world is continuously facing difficult-to-overcome challenges such as coping with changes in customer demands, staying in control while offering mass customization at low costs, or increasing the product range in order to remain competitive. A complex system may be viewed as any system featuring a large number of interacting components whose aggregate activity is nonlinear—not derivable from the summations of the activity of individual entities—and typically exhibits hierarchical self-organization under selective pressures. Understanding complexity, therefore, can help formulate theories that can predict complex manufacturing phenomena, and help derive the tools that can deal with manufacturing complexity [1]. Deriving and exploring theoretical models for measuring complexity is fundamental to understanding manufacturing complexity, a motivation for this paper.

As an advanced mode of production, cellular manufacturing has been widely used in many manufacturing organizations and it can be introduced into the traditional manufacturing firms by means of executing technical reformations. Focusing on cellular manufacturing systems, this paper will analyze its structural and operational complexity.

Shannon [2] was the first to introduce in 1948 the concept of measuring the quantity of information by the means of entropy. For a given manufacturing system, the states of each resource are determined by their technological structure and operational status. Therefore, Karp [3] proposed that the states of manufacturing resources can be carried out the measurement and assessment if the amount of information needed to describe their technological structure and operational status is calculated. Checkland [4] argued that measuring manufacturing complexity provides a useful metric for improvement. He argued that systems with higher complexity have more problems than systems with lower complexity. Drestke [5] used static complexity as a measure of complexity due to the system design, while dynamic complexity was seen as the result of the uncertainties in the system while it is operating. Peter [6] concluded that the meaning of information-theoretic entropy calculated in manufacturing systems was different from its common one. According to Calinescu et al. [7], all the information required for assessing the static entropy model is available from the production order and process plans for individual parts. Fujimoto and Ahmed [8] defined a complexity index for assembling. The index takes the form of entropy in evaluating the assemblability of a product.

With all the documents mentioned above, not only is the meaning of information-theoretic entropy defined in manufacturing systems, but also the entropy models or function is developed by analyzing the relationships among facilities, parts, and tasks in the systems and manufacturing systems are de-

scribed qualitatively and quantitatively. But the researchers have emphasized theoretical analysis while they have neglected the application of information-theoretic entropy in manufacturing systems. Furthermore, the model developed is usually confined to the ideal state. This paper intends to develop operable entropy models of cellular manufacturing systems and an empirical study is used to demonstrate the validity of the proposed methodology.

2. Analysis on the complexity and utility of manufacturing systems

Complexity is subjective in that it is dependent on what manufacturing corporation considers to be its main functional requirements that must be achieved in order to gain a competitive edge. Complexity may be seen as heterogeneous in terms of changeability and variability or disorder in the sense of uncertainty or entropy [9]. The perspectives of complexity may be contested in the context of a property of manufacturing systems, referring to that condition of a manufacturing process which must be understood and measured in order to formulate models and simulations of real complex process-based manufacturing systems for the improvement of conventional manufacturing operations or systems.

2.1 An analysis on the complexity of manufacturing systems

When a new manufacturing facility is set up, it will have been designed to work according to a predetermined level of complexity. However, it is likely that the level of complexity will not have been calculated, and so no one will be aware of its effect on the scheduling and day to day running of the facility.

Let us take a plausible situation. Suppose that the facility has a small number of products and has been designed to operate on a cell-based layout. The equipment is new and well maintained, so it works reliably. Such a system would have a very low structural and dynamic complexity. The system still has a reasonable amount of spare capacity and, during the early days, there are still some problems with the suppliers and customers. However, this does not make life too difficult since the human controllers have enough discretion, whether formally acknowledged or not, to amend the schedule to accommodate these occasional local difficulties.

These are rational responses to complexity and an organization can knowingly and rationally allow these practices to become established in order to be able to deliver flexibly and be reliable to demanding customers who are willing to pay a premium for the extra service this represents. Or, the practices can grow up willy-nilly with the costs and consequences undetected and uncontrolled.

But when more negotiations have to be done with suppliers and customers, the proliferation of possible routes through the facility is hard to keep track of, and exactly where everything is at any one time in the facility is anyone's guess. Deliveries to customers are becoming unreliable. Premiums are being
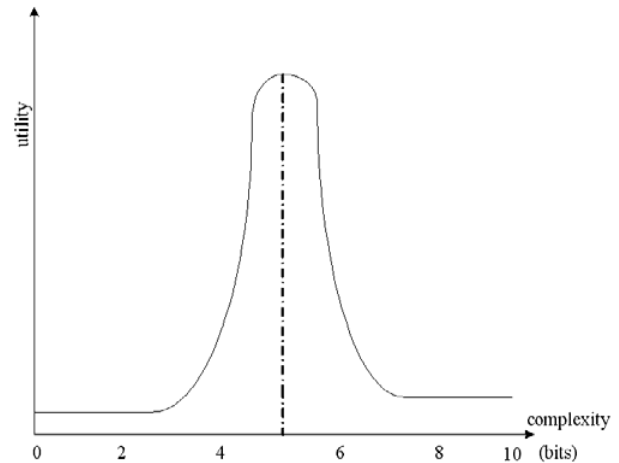

Fig. 1. Sketch of the utility function of complexity for manufacturing systems.

paid to suppliers because of the short lead time orders that are needed to cover the problems. This facility is now becoming too complex. Costs are becoming noticeable. Management is taking an interest in what is going on.

2.2 The complexity and utility function of manufacturing systems

By utility, we mean the perceived value of the complexity. The utility of complexity is likely to be very different for different people, but here we are focusing on the people who know the manufacturing system very well and manage, schedule and control it every day. A utility function for them is sketched in Fig. 1.

The figure may be interpreted as follows. While complexity is low and the system runs according to routine, there is little opportunity to solve problems and be creative. Things might even become a little dull. However, problems come along and people can help one another to solve them. Complexity increases and the utility of the task increases, until it reaches a peak somewhere around the four to six bits level. However, other external uncontrolled factors affect the situation and the facility becomes more complex. Indeed, it becomes too complex and the perceived value of the task starts to go down. This is not a good situation to be in, and so the facility may be simplified to bring the utility up again, and the complexity back down.

So, if we could plot the time history of the complexity of the system, it might look something like Fig. 2. The complexity of the system increases beyond the utility peak and is brought back down again. It drifts back up, only to be brought downwards yet again. However, the underlying trend is always upwards. People, machines and support systems should also become better at dealing with it and keeping it under control.

According to the analysis above, we can see that complexity is a double-edged sword. Too little, and the facility becomes too rigid and inflexible. Too much makes the facility too diffi-
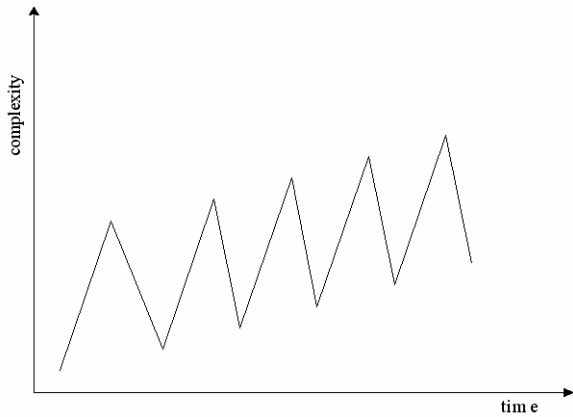
Fig. 2. Trend of the complexity of a manufacturing system.

cult and stressful to control, with too much energy dissipated on solving recurring problems instead of focusing on value-adding activities. Unless complexity is understood, the system could end up focusing its energies on living with solvable problems, instead of gaining advantage through flexible, responsive behavior.

3. The state entropy model of cellular manufacturing systems

The entropy of a random variable is defined in terms of its probability distribution and can be shown as a good measure of randomness or uncertainty. Adapting Suh's definition [9], we define manufacturing information as a logarithmic function of the probability of achieving a set of specified global manufacturing functional requirements. Therefore, we can define static complexity represents the expected amount of information required to define the state of the system for a given period, based on the information in the schedule; dynamic entropy represents the amount of information required to define the state of the system, based on monitoring the system for a given period. According to Shannon's information-theoretic entropy, the more complex a manufacturing operation becomes, the longer it will take parts to get through the system; as a system becomes more complex, it becomes harder to make reliable promises; most complex operation within the system is liable to be the bottleneck. Therefore, the information of the states of manufacturing systems can be quantitatively analyzed through measuring their entropic values.

3.1 Characteristic of cellular manufacturing systems

Cellular manufacturing, based on the theory of group technology, is the mode of production used in manufacturing single or variable products through arranging the manufacturing equipment in the shape of the letter "U" within a work cell. It is an efficient way to cut down the costs, improve the quality of the products and strengthen the manufacturing flexibility. Cellular manufacturing has been widely used in many leading

countries in manufacturing. For example, 60% of manufacturing enterprises in Germany have successfully introduced cellular manufacturing into production [10].

The work cell is the basic unit of cellular manufacturing [11]. It is composed of the manufacturing equipment, operators and equipment for transfer, etc. Usually the manufacturing equipment in the work cell is arranged in the shape of the letter "U." The operators can operate flexibly and efficiently in this U-workshop. Cellular manufacturing systems are different from the traditional manufacturing systems which are progressive in their management structure and are fixed and meticulous in their production structure. Every relatively independent work cell can be a unit of the relatively independent manufacturing systems and several units are managed and coordinated together via the network.

3.2 Static entropy model

In order to deduce the static entropic model of cellular manufacturing systems, we firstly introduce the definition of information entropy.

Given a group of events $X = \{x_1, x_2 \ldots x_n\}$, and the a priori probabilities of the event occurrences $P = \{p_1, p_2 \ldots p_n\}$, the entropy function of X can be expressed as follows:

$$H(X) = -C \sum_{i=1}^{n} p_i \log p_i, \; with \; \log 0 = 0 \tag{1}$$

where C is a constant depending on the base of the logarithm function chosen. If $\log_2$ is selected, $C = 1$ and the unit of entropy is bit. The entropy function described in Eq. (1) has the following characteristics:
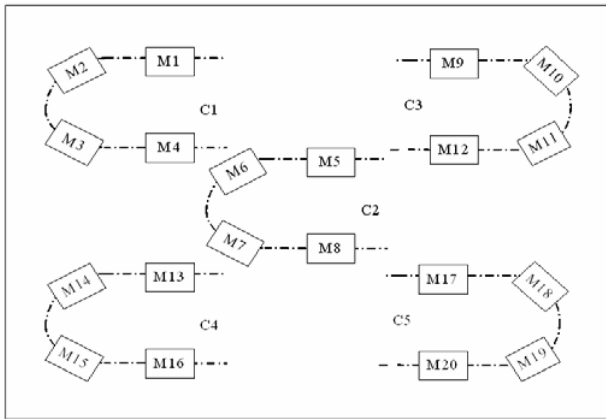
(1) $H = 0$ if and only if all the p_i but one are zero, this one having the value unity. Thus only when we are certain of the outcome, will the entropy be null.

(2) H achieves its maximum, $\log n$ (for $C = 1$), when all the events have equal probability $p_1 = p_2 = \ldots p_n = 1/n$. This situation corresponds to maximum uncertainty.

(3) H does not change if an additional event x_{n+1} is included in the system, with $p_{n+1} = 0$.

According to the definition above, consider a manufacturing system with $r \geq 1$ resources. Within the assumption of stationarity, if the kth resource has s_k possible states, then the static entropy associated with that resource, from Eq. (1), we can deduce static entropy model for resource k :

$$H_S(k) = -\sum_{j=1}^{s_k} p_{kj} \, lb \, p_{kj} \tag{2}$$

where P_{kj} represents the probability of state j of resource k.

Due to the properties of entropy, and assuming that the events at one resource are independent of the events at any other resource, then the expected amount of information for all resources r within the facility is calculated as

Fig. 4. The cellular arrangement of machines in a job shop.

Fig. 3. The free arrangement of machines in a job shop.

expected to be observed in the operational stage. Taking a discrete manufacturing system for example, if we only pay attention to the facilities' load, then the facilities' states can be defined as "running," "set-up," "maintenances" and "idle." But if we care more about the specific production, we can further define each facility's states as "running product *A*," "running product *B*," "set-up product *A*," "idle," etc. Because the technological structure of manufacturing systems essentially determines possible states of the resources in manufacturing systems, it also determines the static entropy of the systems.

3.3 Dynamic entropy model

The underlying motivation for a definition and measure of dynamic entropy comes from the necessity to identify the system's deviation from the expected behavior, and the difficulty to monitor its status in the operational phase in order to gain this information. In this section, the information-theoretic formula for dynamic entropy model is revisited and discussed.

Dynamic entropy is related to the monitoring of planned and unplanned events. It captures various aspects of manufacturing systems such as size, variety, concurrency, objectives, information, variability and uncertainty. Dynamic entropy quantifies the additional level of information required to define the state of the system when it deviates from the expected behavior.

Based on Eq. (1) and Eq. (2), dynamic entropy model for resource *k* can be expressed as follows:

$$H'_D(k) = -\sum_{j=1}^{S'_k} p'_{kj} \, \mathrm{lb}\, p'_{kj} \tag{4}$$

where S'_k denotes the actual number of states of resource *k* in running, and P'_{kj} denotes the probability of state *j* of resource *k* in the operational.

Based on Eq. (3), dynamic entropy model for all resources *r* can be reformulated as:

$$H_D = -\sum_{k=1}^{r} \sum_{j=1}^{s_k'} p'_{kj} \, \mathrm{lb}\, p'_{kj} . \tag{5}$$

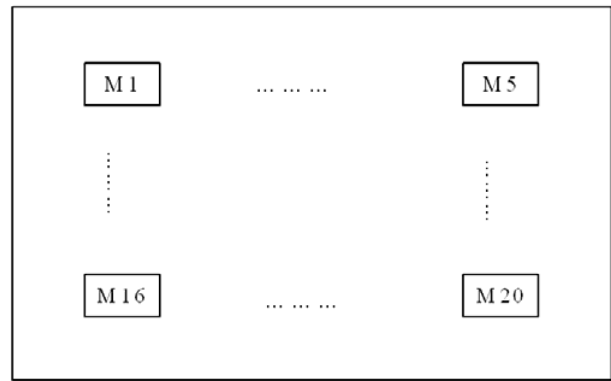The dynamic entropy model given in Eq. (5) is equivalent

to the static entropy model, but in this case, the probabilities are those that are actually measured in practice, rather than those estimated from the schedule. This is denoted by the use of p' rather than *p*. Similarly, the number of states s_k' that are actually observed on each resource will differ from that in the schedule, since unscheduled states such as break down and awaiting resources, may occur.

4. An empirical study

In order to explain the process of calculation of the state entropy models constructed and the degree of complexity decreased in a manufacturing system, this paper takes a mechanical job shop that actualizes cellular manufacturing as an empirical study.

There are 20 machines of four different types making 200 products in some manufacturing facility. Each product needs to pass through one of each type of machine. Before actualized cellular manufacturing, the machines of job shop are arranged by a free mode, which is shown in Fig. 3. (Although the machines are assigned numbers, they are not arranged according to the order of the number.)

Because the job shop is organized completely flexibly, the probability of any machine being in any state (including scheduled idle), P_{ij}, is 1/201. The number of possible scheduled states on each machine is 201, one state for each product and one for scheduled idle. For the convenience of calculation, we suppose manufacturing systems obey the schedule, so the static entropy and the dynamic entropy in a manufacturing system are equal and we only need to calculate one of them.

According to Eq. (3) and Eq. (5), the state entropy of manufacturing systems before actualized cellular manufacturing can be calculated as follows:

$$H_D = -\sum_{k=1}^{r} \sum_{j=1}^{s_k'} p'_{kj} \, \mathrm{lb}\, p'_{kj} = -\sum_{k=1}^{20} \sum_{j=1}^{201} \frac{1}{201} \mathrm{lb} \frac{1}{201} = 153\, bits .$$

Cellular manufacturing is actualized in order to improve the efficiency of production in the job shop. All machines are

composed of five work cells $(C_1—C_5)$ and every cell has four machines $(M_1—M_{20})$. The equipments are arranged in the shape of the letter "U" within a work cell, which is shown in Fig. 4.

The products can now be grouped into five families of 40 products each. So, for each machine, P_{ij}=1/41 and S_i=41. It is no longer necessary to check the status of every machine in the cell, since any one machine in the cell, will be in the same state as all the others. So, instead of checking every machine individually, we only need to check each cell, so $m = 5$.

According to Eq. (3) and Eq. (5), the state entropy of manufacturing systems after actualized cellular manufacturing can be calculated as follows:

$$H_D = -\sum_{k=1}^{r}\sum_{j=1}^{s_k'} p'_{kj}\,\mathrm{lb}\,p'_{kj} = -\sum_{k=1}^{5}\sum_{j=1}^{41} \frac{1}{41}\,\mathrm{lb}\,\frac{1}{41} = 27\,bits .$$

Based on the above result, we can conclude that the state entropy of manufacturing systems is greatly decreased after actualized cellular manufacturing and the degree of decrease is:

$$\frac{H_d - H_d'}{H_d'}\times 100\% = 82\%.$$

That means we will much easily and effectively control a manufacturing system.

5. Analysis and discussion

From the empirical study in Section 3, we have a conclusion that it is a very important problem to deal with an overly complex system. There are two main responses can be depended on. One is to simplify the system again and the other is to install an IT system to manage the complexity of manufacturing systems.

For the first way, this would mean removing much of the discretionary scheduling and decision-making. Products are grouped into families and assigned to cells, with very limited discretion allowed on allocating occasional job elsewhere. Maintenance is re-introduced so that machines perform reliably again. Lot sizes are rationalized to match current market trends. Realistic schedules are introduced and the facility encouraged and helped to stick to them again. Costs come under control.

The other option is to install an IT system to manage the complexity. This will be a costly option, so the benefits need to be carefully understood. If the extra complexity and flexibility that the IT system can provide will pay for themselves in terms of increased charges to customers, then it may be worth doing. The causes of the complexity will still be there, and they may be more difficult to remove once the IT system is installed, since the money and motivation to do so will no longer be there.

6. Analysis and discussion

An important problem needed to be solved is how to deal with the ever-increasing complexity of structure and operation of a manufacturing system as well as diversification and individuation of customer's requests for current manufacturing organizations. Taking cellular manufacturing systems as research subject, this paper divides the amount of information needed to describe its states into the static entropy and the dynamic entropy, on the basis of analysis of recent progress of information-theoretic entropy applied in manufacturing systems, and then develops the static entropy and the dynamic entropy models of manufacturing systems respectively.

The complexity of a manufacturing system that is under human control will rise over time, due to the combination of a more complex and uncontrollable external environment and the human tendency to establish informal customs and practices to solve problems. Unless action is taken from time to time to reduce the complexity, it may rise to a level that is overwhelming. Action may be taken to manage or reduce the complexity, but this need to be done in a disciplined way. Complexity is costly and time-consuming, but extra advantages can be had from coping with complexity.

Acknowledgment

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